Decibel measurements can also be calculated using voltage instead of power assuming that voltages are applied across a constant resistance. Substituting Ohm's law for the power terms yields the following definition:

dB =
$$10\log_{10} \frac{\frac{V_{OUT}^2}{R}}{\frac{V_{IN}^2}{R}} = \frac{V_{OUT}^2}{V_{IN}^2} = 20\log_{10} \frac{V_{OUT}}{V_{IN}}$$

There are some common decibel levels with inherent reference points used to indicate the strength of an audio or radio signal that transform the decibel into an absolute measurement. The unit dBm is commonly seen in audio applications, where 0 dBm is one milliwatt. Therefore, 8 dBm is 6.3 mW. Radio applications sometimes use the absolute unit dB μ V, where 0 dB μ V is 1 μ V.

Decibel units enable the analysis of signals with very low and very high amplitudes. A typical radio receiver may be sensitive enough to detect signals with -90 to -110 dBm of strength. Trying to work with such small numbers without a logarithmic scale is rather awkward.

A common decibel value that is used in frequency domain analysis is -3 dB, which corresponds to a roughly 50 percent reduction in power through a circuit element (10 log₁₀ 0.5 \approx -3). Because decibels are a logarithmic function, the addition of decibel measurements corresponds to a multiplication of the underlying absolute values. Therefore, -6 dB corresponds to quartering the power through a circuit (0.5 \times 0.5 = -3 dB + -3 dB = -6 dB), and -9 dB corresponds to approximately 12.5 percent of the power passing through.

Frequency domain analysis takes into account the real and imaginary components of impedance to form the complex number expression for impedance, Z = R + jX, as already discussed. When combined with Ohm's law, currents and voltages with both real and imaginary components result from the impedance being a complex number. It is often desirable to calculate the magnitude of such complex currents and voltages to determine the peak values in a circuit. They are peak values, rather than static, because AC signals are time varying. If the real and imaginary components of an impedance, current, or voltage are plotted on a Cartesian grid as done in Fig. 12.17, the magnitude of their resulting vector can be obtained according to the Pythagorean theorem's statement of the relationship between legs of a right triangle: the square of the length of the hypotenuse (c) equals the sum of the squares of the two other legs (a and b). Therefore,

$$c = \sqrt{a^2 + b^2}$$



FIGURE 12.17 Finding the magnitude of an impedance.

In this example, a resistor and inductor are placed in series with a resulting impedance, $Z = R + j2\pi fL$. The magnitude is expressed as follows:

$$|Z| = \sqrt{R^2 + (2\pi fL)^2}$$

12.9 LOWPASS AND HIGHPASS FILTERS

Filtering is the general process of attenuating the energy of a certain range of frequencies while passing a desired range with little or no attenuation. The noise filtering examples discussed previously are designated *lowpass* filters, because they are designed to pass lower frequencies while attenuating higher frequencies above a certain threshold. They are also *passive* filters, because the circuits are constructed entirely of passive components (resistors, inductors, and capacitors) without any active components (e.g., transistors) to provide amplification. Passive filters are very practical because of their simplicity and small size. They are suitable for applications wherein the signal that is desired to pass through the filter has sufficient amplitude to be used after the filter. In situations in which very weak signals are involved (e.g., tuning a radio signal), amplification may be necessary before, after, or within the filter. Such filters are termed *active* filters. Whether a filter is active or passive, the underlying analysis of how desired frequencies are passed and undesired frequencies are attenuated remains the same.



FIGURE 12.18 First-order RC lowpass filter.

A filter's frequency response can be determined by combining basic circuit principles with the frequencydependent impedance characteristics of capacitors and inductors. The most basic lowpass filter is a series resistor and shunt capacitor as shown in Fig. 12.18. This is referred to as a first-order lowpass filter, because the circuit contains a single AC element, a capacitor. Since a capacitor's impedance drops as the frequency increases, higher frequencies are short-circuited to ground.

The filter's output voltage can be calculated by combining the impedances of each element into a single voltage-divider expression,

$$V_{OUT} = V_{IN} \frac{Z_C}{Z_C + Z_R} = V_{IN} \frac{\frac{1}{2\pi fC}}{\frac{1}{\frac{1}{2\pi fC} + R}} = V_{IN} \frac{1}{\frac{1}{1 + 2\pi fRC}}$$

Rather than continuing to explicitly reference the input and output voltages, it is common to refer to the filter's *transfer function*, or *gain* (A), which is the relationship between input and output voltage. The gain is obtained by simply dividing both sides of the equation by the input voltage.

$$A_{FILTER} = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + 2\pi f RC}$$

A lowpass filter is considered to be effective starting at a certain *cutoff frequency* (f_c), the frequency at which its power gain in halved. This is also called the *half-power point*. Recall that a decibel level is calculated as 10 log₁₀ A_{POWER} . Therefore, the filter's gain declines by 10 log₁₀ 0.5 = -3